TEMPERATURE-DEPENDENT RESISTIVITY OF A FLAT CONDUCTOR AND THE TEMPERATURE, CURRENT, AND ELECTRIC FIELD PATTERNS

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Characteristic physical quantities have been determined as functions of the nonlinearity criterion and the frequency criterion for a flat conductor heated by a current having stationary and current distributions but temperature-dependent conductivity.

It is necessary to perform local corrections for the temperature dependence of the specific resistance in a current-heated conductor when calculating the temperature and current distributions for high temperatures and high heat fluxes.

The steady-state temperature and field distributions in a flat conductor $x \leq 1$ heated by a monochromatic current may be determined from the following system of nonlinear differential equations in the presence of surface heat loss (the temperature and the tangential monochromatic electric vector are given at the surfaces), in particular, when the specific resistance is temperature-dependent:

$$\rho t'' = -\varepsilon^2, \qquad \rho e'' = in^2 e; \qquad t(1) = t'(0) = e'(0) = 0, \qquad e(1) = 1, \tag{1}$$

where the current density is $j = e/\rho$; the specific feature here is $\rho(t)$. We assume the linear dependence $\rho(t) = kt + 1$ usual for solid conductors. The purpose here is to determine the effects of $\rho(t)$, which are usually neglected, i.e., the quantitative effect of the nonlinearity criterion k, which influences various characteristic physical quantities (functionals defined by criteria) and which can be derived from (1) as coordinate distributions $t(x)$, $e(x)$, and $j(x)$.

System (1) cannot be integrated in quadratures and has been interpreted numerically by matrix fitting by computer with series of values for the criteria n and k and the ranges $0 \leq n \leq 1.57$, $0 \leq k \leq 10^6$; some of the results of major interest are shown in Figs. 1 and 2. The maximum value of $t(0)$ in the median plane is shown as a function of k (argument) for various values of n (parameter), as is the mean conductor temperature $\langle t \rangle$, the total relative change in specific resistance $\delta\rho = kt(0)$ (absent in the linear case, $k = 0$), and the minimal values for the electric field $\varepsilon(0)$, current density $\iota(0)$, and phase $\varphi(0)$ in the mean plane $x = 0$. The quantities $t(0)$ and $\langle t \rangle$ characterize the nonuniformity in the heating; $\varepsilon(0)$ represents the penetration of the external electric field into the conductor (field skin effect), while $\iota(0)$ represents the uniformity in the current distribution in the conductor (current skin effect, which coincides with the field skin effect for $k = 0$, and $\varphi(0)$ represents the phase shift of the electric field at depth and the shift in the current density by comparison with the boundary values.

All these quantities as functions of n or n^2 have turning points with zero derivatives at $n = 0$; they are all decreasing functions of n, and their n dependence becomes weaker as k increases. If k is sufficiently large, $kt(0)$, $\langle t \rangle$, $\delta\rho$ and $\iota(0)$ are virtually independent of n and are functions of k alone. As k increases, the values of $t(0)$, $\langle t \rangle$ and $\iota(0)$ decrease without limit (tend to zero); $\delta\rho$, $\varepsilon(0)$ and $\varphi(0)$ increase, the increases being without bound, tending to 1, and tending to 0 (while remaining negative), respectively.

Theoretical analysis of (1) for very large values of the nonlinearity criterion ($k \gg \max(1; n^4)$) give the following asymptotic relations:

$$t(0) \simeq \sqrt{\frac{2}{\pi k}}, \qquad \langle t \rangle \simeq \frac{1}{\sqrt{\pi k}}, \qquad \delta\rho \simeq \sqrt{\frac{2}{\pi} k}; \tag{2}$$

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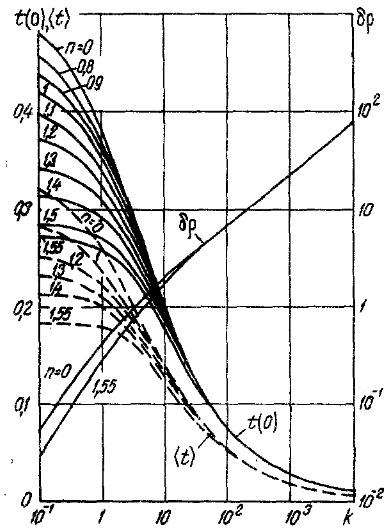

Fig. 1

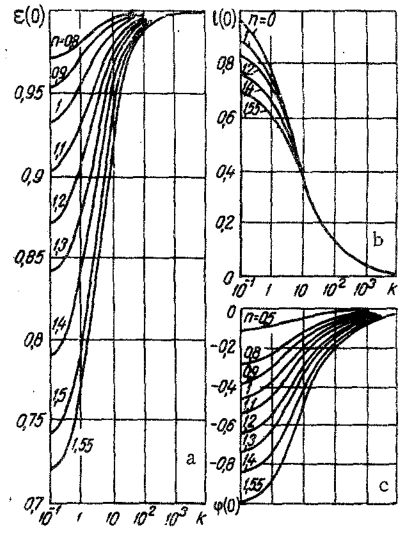

Fig. 2

Fig. 1. Maximum temperature t(0) (in mean plane $x = 0$), mean temperature $\langle t \rangle$ of conductor, and total relative change $\delta\rho$ in specific resistance as functions of k for various n.

Fig. 2. Minimum values in mean plane $x = 0$ of conductor: a) $\varepsilon(0)$; b) $\iota(0)$; c) $\varphi(0)$ as functions of k for various n.

$$\varepsilon(0) \simeq 1 - \left(\frac{1}{2} - \frac{1}{\pi}\right)\frac{n^4}{k}, \quad \iota(0) \simeq \sqrt{\frac{\pi}{2k}}, \quad \varphi(0) \simeq -n^2\sqrt{\frac{2}{\pi k}} \tag{3}$$

These agree with results of numerical integration of (1) for large k.

NOTATION

ω is the angular frequency of current (field);

λ, μ, α are the thermal conductivity, absolute magnetic permeability, and temperature coefficient of specific resistance of conductor;

ρ_0 is the specific resistance near surface (at zero temperature);

$\vec{e}_0$ is the amplitude of tangential electric vector at surface;

$n \equiv a\sqrt{\mu\omega/\rho_0}$ and $k \equiv [(e_0 a)^2/\lambda\rho_0] \cdot [\alpha/(2-\delta_{S0})]$ are the controlling criteria (frequency criterion and nonlinearity criterion, respectively);

$\delta_{S0} \begin{cases} 1, & n = 0\ (\omega = 0); \\ 0, & n \neq 0; \end{cases}$ x, t, $\rho = \rho(t)$,

$e \equiv \varepsilon \cdot \exp(i\varphi)$, $j \equiv \iota \cdot \exp(i\varphi)$,

$(\varepsilon \equiv |e|, \iota \equiv |j|, \varphi \equiv \arg e = \arg j)$ are the coordinate ($|x| \leq 1$), temperature of surface $|x| \leq 1$ of a planar conductor, specific resistance, complex electric field, and current density amplitude (everywhere parallel to $\vec{e}_0$) as referred to half-thickness a of the planar conductor and to $(e_0 a)^2/\lambda\rho_0(2-\delta_{S0})$, ρ_0, e_0, e_0/ρ_0, respectively;

$\langle t \rangle \equiv \int_0^1 t\,dx$ is the mean temperature of conductor;

$\delta\rho \equiv \rho[t(0)] - 1$ is the total relative change in specific resistance of conductor.